Redistribution of Magnetic Helicity at the Sun

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Abstract. Evolution of magnetic loops associated with coronal mass ejections involves a redistribution of solar magnetic helicity. The conservation of magnetic helicity then implies that ejection of helicity into the solar wind has to be accompanied by encapture of an equal amount of helicity of opposite sign at the Sun. A simple model is constructed which includes this effect and other MHD effects on the evolution of magnetic helicity. The model indicates that the magnetic helicity of each hemisphere of the Sun oscillates about a mean with the half-period of the solar cycle (11 years). The helicity in a given hemisphere does not change sign from one 11 year period to the next.

Introduction

The magnetic field structure plays an important role in the transfer of energy between the solar photosphere, the corolla) and the solar wind. Dynamical changes in the field structure allow the release of stored magnetic energy during flares, microflares, and coronal mass ejections. Magnetic structures are typically three-dimensional and expected to be topologically nontrivial. For example, Hα and X-ray observations provided by the Solar Maximum Mission and YOHKOH imply that twisted and braided magnetic flux tubes constitute a basic structural element of the solar atmosphere [110USC and Berger, 1987, Shibata et al., 1992]. Interplanetary clouds associated with coronal mass ejection's (CMEs) in interplanetary space reveal helical magnetic fields [Lepping et al., 1990]. Smith and Bieber (1993) noted that the interplanetary magnetic field at 1AU is more tightly wound than would be expected for the Parker's spiral and interpreted this "overwinding" as an escape of the helical toroidal magnetic field from the Sun. Rust (1994) pointed out that the sign of magnetic field twist (called "the chirality") of a cloud corresponds to the predominant chirality of filaments in the solar hemisphere in which that cloud originated. The clouds' chirality agrees with that deduced from 11a observations [Martin et al., 1994]: clouds with the left-handed magnetic configuration seen to come from the northern hemisphere and right-handed ones come from the south [Rust and Kumar, 1994]. The problem of predicting the sign of magnetic field twist in CMEs and interplanetary magnetic clouds is discussed by McAllister and Martin (1996). To make such predictions some mechanism describing the origin of the helical magnetic field has to be assumed. It has been proposed that helical magnetic configurations originate from three-dimensional magnetic reconnections of extended coronal loops which become CMEs as they disconnect from the Sun [Gosling et al., 1995, see also Crooker et al., 1990].

Magnetic reconnections in the highly conducting solar corona, however, can not result in a magnetic configuration having helicity of only one sign. The reason is that the helica structure of magnetic lines is characterized by a topological invariant - known as the *magnetic helicity* $\int \mathbf{B} \times \mathbf{A} d^3x$, where $\mathbf{B}$ is the magnetic field and A is its vector potential. The helicity invariant is a conserved integral in an ideally conducting fluid [Moffatt, 1978]. The conservation of the magnetic helicity in the highly conducting solar and heliospheric plasmas suggests that, by measuring this quantity in the solar wind, one can obtain valuable information about solar activity. When the magnetic field can be considered as organized into flux tubes, the contribution to the helicity comes from the *twist* and *writhe* (curling of the magnetic field into coils or folds) of the individual flux tubes and *links* between different flux tubes [1 Berger and Field 1984, Wright and Berger, 1 989]. It has been shown that the magnetic helicity can effectively survive the process of magnetic line reconnections in highly conducting plasma [Taylor, 1974, 1 Berger 1984, Ruzmaikin and Akhmetiev 1994]. (Although due to a finite conductivity the helicity is not exactly conserved, its change requires a much longer time than does the change in the magnetic energy.) What happens in reconnections are transformations of the three forms of helicity into each other - for example two linked loops can merge into one with their link being converted into a twist or writhe - with conservation of the total helicity. The creation of helicity of a single sign by reconnections hence would violate the conservation of helicity in highly conducting plasmas. What takes place instead is either a redistribution of magnetic helicity or the creation of equal amounts of helicity of both signs followed by redistribution of the helicity. Thus, to be in agreement with the conservation of Ills.gllctic helicity, the ejection of helicity into interplanetary space has to be accompanied with the ejection of opposite-signed helicity into another volume of space and/or the encapture of an equal amount of helicity of opposite sign at the Sun.

The present paper will illustrate first - taking as an example the structure considered by Gosling et al., 1995- how magnetic reconnection among magnetic loops observed to be rooted in the photosphere can introduce helicity into an erupting magnetic field accompanied with an encapture of helicity by the Sun. Then a simple theoretical model, which includes the effect of helicity encapture and other MHD effects contributing to helicity evolution in the Sun, will be constructed. The model indicates that the magnetic helicity of each hemisphere of the Sun oscillates about a mean with the half-period of the solar Cycle (11 years). It also predicts that the helicity in a given hemisphere dots not change sign from one 11 year period to the next.

Encapture of Magnetic Helicity at the Sun due to Ejections

First, note that the concept of "topology" of a single magnetic line is meaningless. The line has to be linked with another line or twisted around another line. The convenient topological object is a magnetic flux tube which can carry magnetic helicity in the form of twist and writhe. A coiled (writhed) magnetic flux tube can be produced by reconnections between magnetic field lines of two flux tubes that have emerged above the solar surface (see Figure 1, following Gosling et al., 1995). This type of picture, however, does not carry enough information to be able to state whether or not the flux rope has magnetic helicity. Magnetic helicity is a non-local quantity - because the vector potential at a given point is determined by the magnetic field from all points. In this particular example, depending on how the magnetic lines rooted into the Sun are connected with each other under the surface, the initial magnetic configuration may or may not have helicity. Note that a coil (writhe) on a closed flux tube is equivalent to helicity [Berger and Field 1984, Wright and Berger, 1989]. In order to visualize this, one can consider two field lines in a closed figure-8 flux tube and try to drag one line away from the other (Figure 2).

Conservation of magnetic helicity implies that if helical magnetic fields escape into the solar wind, an equal amount of magnetic helicity of opposite sign has to be captured at the Sun. The third part of Figure 1 gives an illustration what happens beneath the solar Surface.

A Model of Evolution of Magnetic Helicity

The same conservation law inhibits the creation of magnetic helicity inside the Sun. The only effective way of magnetic helicity production is again based on the idea of the helicity redistribution. Such magnetic helicity production in a fully developed kinetically helical turbulence has been demonstrated in numerical simulations [Pouquet et al., 1978]. In these simulations the system was excited by injection of kinetic energy and kinetic helicity (which is not a conserved quantity) at some basic scale. A small amount of magnetic energy was also given initially. Then, as time increased, the magnetic energy grew at progressively larger scales. No magnetic helicity was given initially. However, the magnetic helicity appeared as time increased. This helicity had the same sign as the kinetic helicity at the basic scale and the opposite sign at larger scales. Thus, magnetic helicity can be produced at some scale with the simultaneous production of opposite signed magnetic helicity at another, larger or smaller, scale. In the simple model constructed below only the evolution of the large-scale part of the magnetic helicity is considered. The role of the accompanying small-scale helicity and consequences of the helicity separation will be discussed in a separate paper.

Consider here a mean axisymmetric magnetic field averaged over small scales and times, say over the supergranulation scale. The magnetic field is split into its toroidal (azimuthal) component $B = B_\phi$ and poloidal components B_r, B_θ defined by the azimuthal component of the vector potential $A = A_\phi$

$$B_r = \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A)}{\partial\theta}, \quad B_\theta = -\frac{1}{r}\frac{\partial(rA)}{\partial r}$$

Consider the magnetic helicity in one hemisphere of the convective zone

$$H = 2\pi\int_{R_{cz}}^{R_\odot}\int_0^{\pi/2} ABr\sin\theta\, d\theta\, dr$$

where R_{cz} is the position of the bottom of the solar convective zone, and $R_\odot$ is the radius of the Sun. The sign of H and its change during the solar cycle can be determined from symmetry considerations: Hale's law says that the azimuthal component B has opposite signs in the northern and southern hemispheres of the Sun and that the sign relation (+,-) changes into (-,+) in a half solar cycle, i.e. every years. The radial component also has opposite sign in the two hemispheres and its sign oscillates with the 1 year period. It follows that the vector potential A has the same sign in the northern and southern solar hemispheres. However this sign changes every 11 years. Thus the magnetic helicity has opposite signs in the northern and southern hemispheres of the Sun and this sign relation does not change from cycle to cycle. It will be shown later that the amplitude of magnetic helicity oscillates with an 11 year period.

The evolution of the mean magnetic field in the solar convective zone is described in spherical coordinates r, θ, ϕ by equations

$$\frac{\partial A(t,r,\theta)}{\partial t} = \alpha B \cdot \beta(\nabla^2 \quad \mathrm{cosec}^2\theta)A \tag{1}$$

$$\frac{\partial B(t,r,\theta)}{\partial t} = (\nabla\Omega \times \nabla)_\phi r \sin\theta A \quad \beta(\nabla^2 \quad \mathrm{cosec}^2\theta)B \tag{2}$$

where α is proportional to the kinetic helicity of convective motions. β is the diffusivity which includes also a turbulent diffusivity, Ω is the angular velocity in the convective zone, and B is the toroidal component of the magnetic field (see for example Zeldovich et al., 1984).

The evolutionary equation for the magnetic helicity can be obtained from Eqs. (1, 2) by multiplying the first of these equations by B, the second by A, summing and integrating the resulting equations over a hemisphere. To simplify the equation it is assumed that the effect of the diffusion terms can be reduced to a turbulent dissipation with a characteristic time τ_T. Then the equation becomes

$$\frac{\partial H(t)}{\partial t} = \int \alpha B^2 d^3r \cdot \frac{1}{2}\int (\nabla\Omega \times \nabla)_\phi r sin\theta A^2 d^3r \quad \frac{H}{\tau_T} \qquad (3$$

where the integrals are taken over the convective zone in one hemisphere. Let us make an additional simplification by assuming that the first term in the right side of this equation is larger that the second one. This assumption is based on the fact that the intensity of the toroidal component of the solar magnetic field is much higher than the intensity of the poloidal field. However the relative magnitudes of the kinetic helicity and the differential rotation are poorly known, so that this assumption needs to be verified by further studies. Now complement this equation with the encapture effect discussed in the previous section. Because the appearance of magnetic loops on the surface of the Sun is mostly due to the emergence of the subphotospheric azimuthal magnetic field let us assume that this effect can be accounted for by a term of the form $-2pE_B$, where $E_B = 1/2 \int B^2 d^3r$ is the energy of the azimuthal magnetic field and p is a normalized rate of the magnetic helicity escape. Thus it is assumed that magnetic helicity is ejected into the solar wind at the rate proportional to the rate of flux emergence which is in turn proportional to the intensity of the toroidal field.

For α constant in one hemisphere the resulting equation for the evolution of the magnetic helicity now takes the form

$$\frac{\partial H(t)}{\partial t} = 2(\alpha - p)E_B(t) - \frac{H(t)}{\tau_T} \tag{4}$$

Note that in the standard solar dynamo models α has different signs in northern and southern hemispheres of the Sun.

To analyze the character of the solutions of Eq. (4) one has to specify the time dependence of the azimuthal field energy. The simplest assumption is a solar cyclic dependence, $E_B = E_0 \cos^2 \omega t$ where $w = 2\pi/22$ years. In this case Eq. (3) has an analytical solution

$$H(t) = C\exp(-t/\tau_T) + (\alpha - p)E_0\left(1 + \frac{\sin(2\omega t + \phi)}{\sqrt{1 + 4\omega^2\tau_T^2}}\right), \quad \phi = \arctan\frac{1}{2\omega t} \tag{5}$$

After a few diffusion times the first term vanishes, and the magnetic helicity will behave as a periodic function with a period equal to half of the solar cycle period oscillating around the mean value $(\alpha - p)E_0$. The sign Of the magnetic helicity is defined by the sign of the difference $(\alpha - p)$ and according to Eq. (5), does not change from cycle to cycle.

Theoretical arguments (see for example Ch. 11 Zeldovich et al., 1 984) predict that α is positive in the northern hemisphere and negative in the southern hemisphere. If one assumes that the rate of helicity ejection p is less than its production by the kinetic helicity in the solar convection zone, the mean magnetic helicity, according to Eq. (5), would have the same sign as α. These signs seem to be in agreement with the signs of predominant chirality in the solar hemispheres (see Introduction). Note, for the sake of clarity, that positive helicity corresponds to counter-clockwise, i .e. left,-llmldd chirality.

Discussion

The relative amplitude of the helicity oscillations in Eq. 5) is determined by the dimensionless value of

$$2\omega\tau_T = \frac{4n}{22\text{years}}\tau_T \approx \tau_T/2(\text{years})$$

The turbulent diffusivity in the solar convective zone has been estimated to be about $\nu_T = 5 \times 10^{12}$ cm^2/sec [Zeldovich et al., 1984] which gives an estimate $\tau_T = h^2/\nu_T \approx 2$ year. Here $h = 2 \times 10^{10}$ cm is the size of the convective zone. Thus $2\omega\tau_T \approx 1$ and the value of $H/(\alpha - p)E_0$ oscillates between $1 + 1/\sqrt{2} \approx 1.7$ and $1 - 1/\sqrt{2} \approx 0.3$. This effect ought to be observable in the solar wind.

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Figure Captions

Figure 1. A sketch demonstrating how two flux tubes can reconnect above the solar surface to create a coiled flux tube above the surface of the Sun. The parts of the flux tubes above the solar surface are drawn with thick lines. The parts below the solar surface are drawn with thin lines. The coil above the surface has a positive writhe, the coil inside the Sun has a negative writhe so that the resulting magnetic helicity is zero.

Figure 2. The illustration of the fact that the writhe of a flux tube is equivalent the helicity provided by links of magnetic lines. Figure 8-form flux tube has a non-negative helicity. If we will drag one of the field lines (say the one shown thinner) away from the figure-8 we will end with two linked field lines.

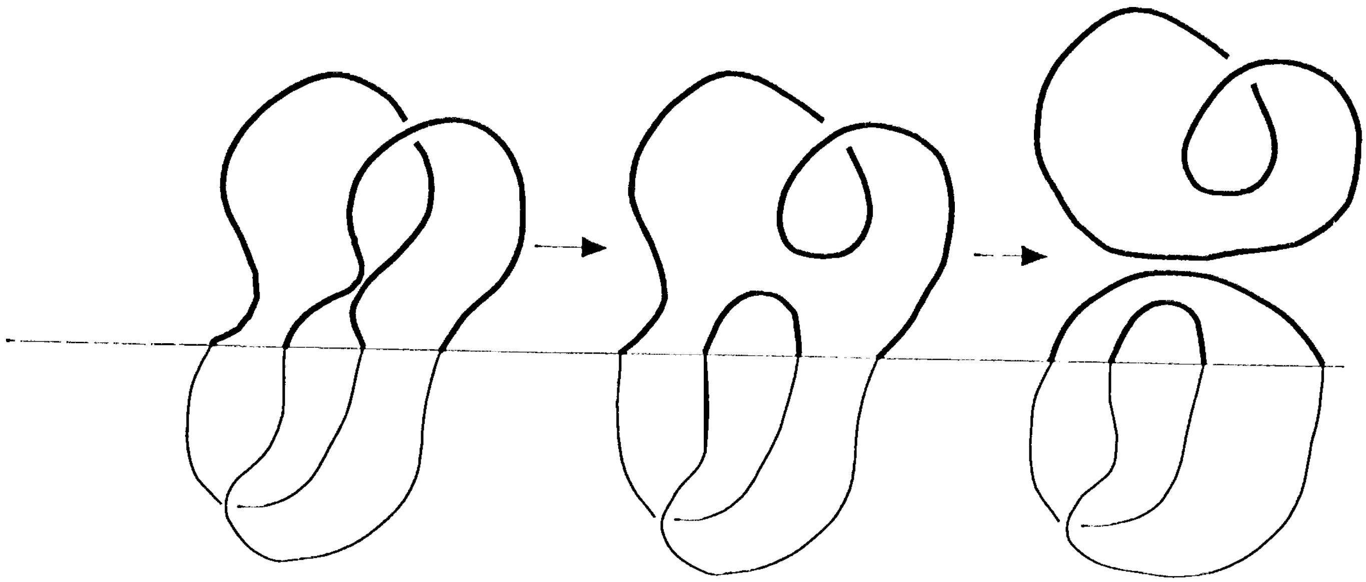

Fig. 1
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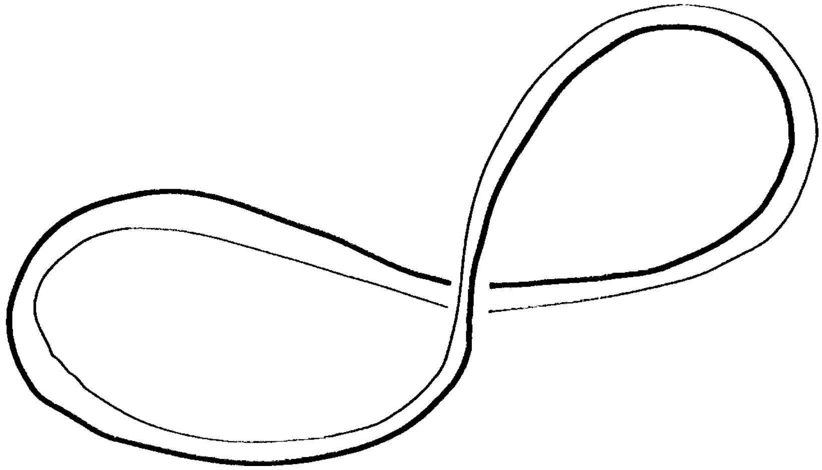

Fig. 2

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